A FINSLER DIFFERENTIAL-GEOMETRIC DESCRIPTION OF DEFORMABLE SOLIDS WITH MICROSTRUCTURE

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Abstract

Finsler differential geometry is used to construct a new theory of mechanics of solid materials broad enough to encompass various metals and ceramics, both crystalline and amorphous. The general model accounts for finite deformation [1,2], nonlinear elasticity [1], and microstructural heterogeneities such as structural defects, including cracks, shear bands, twins, and/or dislocations [1]. The general kinematic structure of the theory includes macroscopic and microscopic displacement fields; the latter are represented by a state vector of (pseudo-)Finsler space. A fundamental tensor is introduced, leading to linear and nonlinear connections. An appropriate deformation gradient is newly derived via delta-differentiation of motion. Euler-Lagrange equations are derived for quasi-static equilibrium, demonstrating importance of the trace of Cartan's tensor. The new theory is invoked to describe physical problems of tensile fracture, shear localization, and cavitation. Finsler character of the metric is achieved via Weyl-type rescaling, i.e., a conformal transformation. Analytical solutions are compared to predictions of Griffith's fracture mechanics and phase field models. The new pseudo-Finsler theory is shown to encompass classical approaches: phase field solutions can be recovered when simplifying assumptions are imposed, as can other descriptions framed in Riemannian [1,2] as opposed to either Finslerian or locally Minkowskian spaces. The present solutions offer new physical insight into coupling of microscopic dilatation with fracture or slip. An increase in the Weyl parameter correlates with an increase in peak strength and energy, physically indicative of increasing slip resistance or crack surface friction coupled to microscopic dilatation. Such effects arise naturally from solution of governing equations derived via fundamental mathematical physics, without resort to ad-hoc extensions of prior theories or exercises in parameter fitting.

References

[1] Clayton, J.D., 2011. *Nonlinear Mechanics of Crystals*. Springer, Dordrecht.

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